Abstract:

The current study investigates the interaction between multiple fiber-shaped piezoelectric sensor (the inhomogeneity ) and a nearby crack. The piezoelectric sensor is embedded in a non-piezoelectric elastic matrix which contains a crack. The matrix is subjected to a far field in-plane tension and the fibers are applied by anti-plane electric loading at infinity. The interaction between the fibers and the crack is partially decoupled through a decomposition process. During the solution procedure, the crack is simulated as a continuous distribution of edge dislocations. By using the solution of an edge dislocation near multiple inclusions in elastic media as the Green function, the problem is formulated into a set of singular integral equations, which are solved by numerical method. The stress intensity factors are derived in terms of the asymptotic values of the dislocation density functions evaluated from the integral equations. Numerical examples are given for various material constants combinations and geometric parameters.

1. Introduction

Piezoelectric materials are widely used in more and more sophisticated engineering fields because of their intrinsic electromechanical coupling behavior. Piezoelectric ceramics based on ferroelectric crystals such as lead zirconate titanate (PZT) and barium titanate (BaTiO3) are employed as electromechanical sensors, transducers and actuators [J. Herbert, Ferroelectric Transducers and Sensors, Gordon and Breach Science Publishers, New York, 1982.][ I. Chopra, Smart structures and integrated systems, SPIE Proceedings, Bellingham, Washington, 1995.]. A considerable amount of attention has been given to the practical importance of piezoelectric materials, such as Deeg [W.F. Deeg, The analysis of dislocation, crack, and inclusion problems in piezoelectric solids, Ph.D. thesis, Stanford University, 1980.], Pak [Y.E. Pak, Linear electro-elastic fracture mechanics of piezoelectric materials, Int. J. Fract. 54 (1992) 79–100.], Suo et al. [Z. Suo, C.-M. Kuo, D.M. Barnett, J.R. Willis, Fracture mechanics for piezoelectric ceramics, J. Mech. Phys.

Solids 40 (1992) 739–765.], etc. Inhomogeneities either unwanted or deliberately introduced in the materials can dramatically change their mechanical properties. Therefore the study of inhomogeneities (or inclusions) has received a considerable amount of attention (see, for example [Zhou K, Keer LM,Wang QJ. Semi-analytic solution for multiple interacting three-dimensional inhomogeneous inclusions of arbitrary shape in an infinite space. Int J Numer Meth Eng 2011;87:617–38.][ Zhou K,Keer LM,Wang QJ,AiX, Sawamiphakdi K,GlawsP, etal. Interaction of multiple inhomogeneous inclusions beneath a surface. Comput Methods Appl Mech Eng 2012 ; 217–220: 25–33.][ Wang X, Zhou K. Three-phase piezoelectric inclusions of arbitrary shape with internal uniform electroelastic field. Int J Eng Sci 2013;63:23–9.][ Wang X, Zhou K. Long-range interaction of a line dislocation with multiple multicoated inclusions of arbitrary shape. Acta Mech2013;224:63–70.]). When piezoelectric materials are used as sensors, they are usually embedded in non-piezoelectric materials. Due to the electro-mechanical interaction between the piezoelectric sensor and the surrounding matrix material, the properties of the surrounding material may influence the response of the sensor to external loading. With the application of this phenomenon, the sensor can be used to detect possible defects in the surrounding material. It is desirable to understand the interaction between the piezoelectric sensor and the near-by defects in the matrix for defect-sensing purpose.

The interaction problem between cracks and inclusions in materials has been an important topic in the literature. For instance about traditional crack-inhomogeneity interaction problems in pure elastic media, the interaction between a crack and a circular inclusion in a sheet under tension was studied by Tamate [Tamate O. The effect of a circular inclusion on the stresses around a line crack in a sheet under tension . Int J Fract 1968; 4: 2 57–65.]] and an exact solution of the stress field at the neighborhood of the inclusion was obtained. Atkinson [C. Atkinson, The interaction between a crack and an inclusion, Int. J. Eng. Sci. 10 (1972) 127–136.] first studied the interaction problem between a crack and an inclusion by using numerical method. Erdogan and his coworkers[ Erdogan F, Gupta GD, Ratwani M. Interaction between a circular inclusion and an arbitrarily oriented crack. ASME J Appl Mech 1974; 41:1007–13. ] considered the interaction between an isolated circular inclusion and a line crack embedded in an infinite matrix with the distributed dislocation method. The interaction between an elastic circular inclusion and two symmetrically placed collinear cracks was investigated by Hsu and Shivakumar [Y.C. Hsu, V. Shivakumar, Interaction between an elastic circular inclusion and two symmetrically placed collinear cracks, Int. J. Fract. Mech. 12 (1976) 619-630.]. The distributed dislocation method is an effective tool to solve various kinds of crack problems [Dai DN. Modeling cracks in finite bodies by distributed dislocation dipoles. J Fatigue Fract Eng Mater Struct 2002;25:27–39.] [ Hills DA, Comninou M. Anormally loaded half plane with an edge crack. Int J Solids Struct1985;21:399–410.] [Helsing J. Stress intensity factors for a crack in front of an inclusion. Eng Fract Mech 1999; 64: 245–53.] [Han J, Dhanasekar M. Modeling cracks in arbitrarily shaped finite bodies by distribution of dislocation. Int J Solids Struct 2004;41:399–411.][ Jin X, Keer L.Solution of multiple edge cracks in an elastic half plane . Int J Fract 2006;137:121–37(w).][ Nowell D,Hills DA. Open cracks at or near free of positive radial stresses along the crack line in the edges. J Strain Anal1987, 22:177–85.]. The investigation for a crack near an elliptic inclusion was carried out in terms of the body force method by Nisitanietal. [Nisitani H, Chen DH, Saimoto A. Interaction between an elliptic inclusion and a crack. In: Proceedings of the 1996 fourth international conference on computer-aided assessment and control, Computational Mechanics Inc., Billerica, MA,USA,1996;4:pp.325–32.]. Luo and Chen [Luo HA, Chen Y. Matrix cracking in fiber-reinforced composite materials. ASME J Appl Mech1991;58:846–8.] investigated the matrix cracking in fiber-reinforced composite materials. Liu etal. [Liu Y, Ru CQ, Schiavone P, Mioduchowski A. New phenomena concerning the effect of imperfect bonding on radial matrix cracking in fiber composites. Int J Eng Sci 2001;39:2033–50.] studied the effects of imperfect bonding on stress intensity factors calculated at a radial matrix crack in a fiber composite subjected to various cases of mechanical loading. Xiao and Chen [Xiao ZM, Chen BJ. Stress intensity factor for a Griffith crack interacting with a coated inclusion. Int J Fract 2001;108:193–205.] studied the interaction between a radial matrix crack and a three-phase circular inclusion. Kim and Sudak [Kim K, Sudak LJ. Interaction between a radial matrix crack and a three-phase circular inclusion with imperfect interface in plane elasticity . Int J Fract 2005;131:155–72.] investigated the interaction between a radial matrix crack and a three-phase circular inclusion with imperfect interface in plane elasticity . Patton and Santare [E.M. Patton, M.H. Santare, The effect of a rigid elliptical inclusion on a straight crack, Int. J. Fract. 46 (1990) 71–79.] investigated the effect of a rigid elliptical inclusion on a straight crack. As for such problems in piezoelectric materials, Sosa [H. Sosa, Plane problems in piezoelectric media with defects, Int. J. Solids. Struct. 28 (1991) 491–505.] presented a two-dimensional electroelastic analysis in piezoelectric media with defects. Dunn and Wienecke [M.L. Dunn, H.A. Wienecke, Inclusion and inhomogeneities in transversely isotropic piezoelectric solids, Int. J. Solids. Struct. 34 (1997) 3571–3582.] analyzed the electroelastic field in and around inclusion and inhomogeneities in piezoelectric solids. Qin [Q.H. Qin, Thermoelectroelastic solution for elliptic inclusions and application to crack–inclusion problems, Appl. Math. Model. 25 (2000) 1–23.] obtained the thermoelectroelastic solution for an elliptic piezoelectric inclusion embedded in an infinite matrix and applied the result to solve crack-inclusion problems. Xiao etal. [Z.M. Xiao, J. Bai. On piezoelectric inhomogeneity related problems-part II: a circular piezoelectric inhomogeneity interacting with a nearby crack. International Journal of Engineering Science 1999 37: 961-976] determined the stress field and the stress intensity factor for a Griffith crack located near a piezoelectric inhomogeneity in an infinite non-piezoelectric matrix. However, most of the analytical solutions presented in the literature are restricted to the problem involved in the interaction of one piezoelectric fiber and a crack. In fact, multiple piezoelectric fibers are usually embedded in a non-piezoelectric matrix with a crack, and moreover the interactions between a crack and multiple fibers have to be considered when they are closely arrayed.

**2. Physical problem and formulation**

In a rectangular coordinate system (j=1,2,3) , we consider an in finite elastic matrix containing N cylindrical piezoelectric fibers which are parallel to each other along the  direction . A crack with length 2c contained in the matrix locates at the origin of the coordinate. It is assumed that the matrix is isotropic, while the fibers are transversely isotropic and polarized along the symmetry axis. The matrix is subjected to a far field in-plane uniform tension  and the fibers are loaded by a uniform electric field  in the  direction. The origin of coordinates is taken at the center of the crack. The crack line is along the -axis.and thus perpendicular to the far field tensile stressc. Additionally, assume that all the inclusions are completely bounded to the matrix. The cross-section of the system is shown in Fig.1, where the regions occupied y the matrix and the inclusions are denoted by ‘m’ and ‘f’, respectively, and  represents the radius of any inclusions.

Fig.1. the interaction between N circular piezoelectric fibers and a crack in an infinite elastic matrix.

As the matrix is pure elastic material, there is no mechanical-electric coupling behavior inside the matrix. By employing the superposition principle of elasticity [S.P. Timoshenko, J.N. Goodier, Theory of Elasticity, McGraw-Hill, New York, 1934.], the solution of the present problem can be obtained through the sum of two sub-problems, as shown in Fig. 1. The sub-problem I shown in Fig. 2 is the piezoelectric fibers embedded in the matrix without the crack. For the sub-problem II shown in Fig. 3, the only external loads are the crack surface tractions which are equal in magnitude and opposite in sign to the stresses obtained in the first problem along the line which is the presumed location of the crack. The superposition of sub-problem I and sub-problem II is thus equal to the original problem.

Fig.2.sub-problem I

Fig.3.sub-problem II

In sub-problem II, an edge dislocation interacts with a circular inhomogeneity while in sub-problem II, an array of edge dislocations with unknown densities Bx and By (the subscripts x and y represent different components of the edge dislocations)interacts with a circular inhomogeneity of the same radius.

The sub-problem I has been solved in the ref. [47]. In the matrix (an infinite plane with N circular holes), the complex potentials have the form of

 (1)

where  and  are unknown coefficients,  is the centre point of the rth fiber,  stands for a reference length which may be defined as , and are related to the stress state at infinity:

 (2)

On the other hand, the complex potentials inside the inclusions  can be expressed as

 (3)

where  and  are unknown coefficients. The unknown coefficients must satisfy the equations as follow



(4)

where









Eqs. (4) and their conjugated equations constitute a set of  linear equations concerning  unknown coefficients . After they are determined, all the complex potentials about the matrix and inclusions are known, and then the stress fields of sub-problem I are related to the complex variables through

 (5)

In the matrix (an infinite plane with N circular holes), the complex potentials have the form of



where  is the location of the dislocation,  and  are unknown coefficients, .

On the other hand, the complex potentials inside the inclusions  can be expressed as













Eqs. (41)–(44) and their conjugated equations constitute a set of  linear equations concerning  unknown coefficients . After they are determined, all the complex potentials about the matrix and inclusions are known, and then the stress fields produced by the dislocation located at can be expressed



Substituting into Eq.() , the shear stress  and the normal stress at produced by a unit glide dislocation at  can be given



where ,represent the regular part of the fundamental solution of the unit glide edge dislocation interacting with the N circular inhomogeneity.

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The stress field of sub-problem II can be obtained through an integral over the fundamental solutions along the crack line.



where ,  and are the dislocation density components at the point .

The single-value condition of displacement vector requires that the density function of the system integral Eq. (6) satisfy the following relation:



The traction free boundary condition given by Eq. (2) requires that the normal and shear stress components along the crack surface are zero, i.e.



Substituting Eq.() into Eq.() , it can be rewrite as



Let , we can transform Eq.( ) into two simple Cauchy-type singular integral equations as follows:



As the crack is embedded in a homogeneous isotropic matrix, both crack tips have the square root singularities. According to Erdogan [F. Erdogan, Mixed boundary-value problems in mechanics, Mechanics Today 4 (1990) 1±85. ], it is apparent that Eq. ( ) are two singular integral equation with index +1, and the fundamental solution of the singular equation are:



where and  are bounded functions in the interval . According to the Cause-Chebyshev solution [F. Erdogan, Mixed boundary-value problems in mechanics, Mechanics Today 4 (1990) 1±85.], Eq. ( ) and ( ) are discreted, and we get a group of  linear algebra equations with the  unknown 



Where



Once the dislocation density functionare evaluated, following Edorgan [F. Erdogan, G.D. Gupta, M. Ratwani, Interaction between a circular inclusion and an arbitrarily oriented crack, ASME J. Appl. Mech. 41 (1974) 1007-1013. ], the stress intensity factors (SIFs) at the both crack tips can be expressed as:



where  is the right crack tip and  is the left crack tip.

**5. Results and discussion**



To verify the rightness of the present results, keep the normalized electric loading, the position of the crack and the length of the crack unchanged, we calculate the variation of the normalized SIFs at the right tip of the crack located near the piezoelectric fiber with the distance factor , where  is the radius of the fiber and  is the distance between the centers of the crack and the fiber. The results given in fig. for the case of ‘one fiber and a crack’ can be compared with the solution provided by Xiao and Bai[]. It is found that the present solution is well agreeable to Xiao’s solution in ref.[].

That shown in fig. is the variation of the normalized SIFs for a crack located at the center of two same piezoelectric fibers with the distance factor, where is the radius of the fiber andis the distance between the centers of the crack and the fiber. It is found that for the soft piezoelectric fiber having lower yang’s model than the matrix, the SIFs in this case is bigger than that in above case. On the contrary, when the inclusion is harder than the matrix, the SIFs in this case is smaller than that in above case. It indicates that whether it is more dangerous or not because of the interaction between the two fibers and the crack dependents on the relative stiffness of the matrix and the inclusion.



To investigate the influence of the electric loading on the crack, the variation of  for the crack located at the center of the two fibers with the normalized electric loading is shown in fig. Due to the electro-mechanical coupling effect of the piezoelectric material, the influence of the electric loading on the crack located between two soft piezoelectric fibers is much greater. It is found that the influence on the normalized SIFs is linear, the SIFs decreases as the electric loading increases. It indicates that only when the electric filed increases in a certain direction does the SIFs increase. The SIFs tends to be infinitely large whenincrease along the negative direction of . Hence, the crack is very dangerous when ranges in the negative value.



That shown in fig. is the variation of at the right tip of a crack located at the center of two different fibers with the normalized radius of the fiber two, where is the radius of the fiber one and is the radius of the fiber two. In this case, we guarantee the distance between the center of the crack and the right edge of the fiber one is equal to that between the center of the crack and the left edge of the fiber two. It is found that for the soft fiber, the variation of increases as the radius of the fiber two increases, but for the hard fiber, it decreases when the radius increases. It indicates that when the soft piezoelectric fiber is replaced by a big one or a hard piezoelectric fiber is replaced by a small one, the crack become dangerous.



That shown in fig. is the variation of  at both tips of a crack located between two same piezoelectric fibers with the distance factor. For the soft piezoelectric fiber, when the right fiber is nearer to the crack (), the SIFs at the right tip is larger than that at the left tip; when the left fiber is nearer to the crack (), the SIFs at the left tip is larger than that at the right tip; it indicate that the tip which is closer to the fiber is much dangerous than another tip. On the contrary, the tip which is closer to the fiber is much safer than another tip for the hard piezoelectric fiber.



That shown in fig. is the variation of  at both tips of a crack located near the triangular array of three piezoelectric fibers with the distance factor. In this case, the triangular array of three piezoelectric fibers is symmetrical about. For the hard piezoelectric fiber, when the crack locates near the fiber one and fiber three, the crack is more dangerous, but for the soft fiber, the crack is closed.



If the triangular array of three piezoelectric fibers is not symmetrical about, the SIFs at the tip of the crack of model II is not equal to 0. The variation of  at both tips of a crack with the distance factoris shown in fig. when the crack is embedded in the triangular array of three fibers, the SIFs of model II is very large and the crack is very dangerous. Fig. shows the variation of  at both tips of a crack with the distance factor. For the hard piezoelectric fiber, when the crack is embedded in the triangular array of three fibers, the value of for the crack is large and the crack is dangerous, but for the soft fiber, the crack is closed.

